

# **Offline Evaluation of Ranking Policies with Click Models**



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#### Motivation

- Recommendations happen everywhere, such as Amazon, Facebook, Adobe Stock, Google Play, Netflix
- Suppose the existing policy  $\pi$





## **Click Models & Estimators**

List estimator [Strehl'2010]

$$\hat{V}_{L}(h) = \frac{1}{|S|} \sum_{(x,A,w) \in S} f(A,w) \min\left\{\frac{h(A|x)}{\hat{\pi}(A|x)}, M\right\}$$

- $\hat{\pi}$ : estimates of the logging policy
- Disadvantages:
  - Have to match the exact lists. The number of lists is extremely large, thus  $\hat{\pi}(A|x)$  is very small



#### **Experiments**

- Yandex dataset
- The dataset is recorded over 27 days
- Each record contains
  - a query ID
  - the day when the query occurs
  - 10 displayed items as a response to the query
  - the corresponding click indicators of each displayed items
- Logged dataset S
  - any records except day d

with the expected CTR  $V(\pi)$ Can we verify a new policy h





satisfies  $V(h) \ge V(\pi)$  based on logged data under policy  $\pi$ ?



- With click-model assumptions, we can build estimators that leverage structures of click feedback
- Document-Based Click Model (DCTR): •  $\overline{w}(a, k | x)$  only depends on item a



- $\hat{\pi}$  is the empirical distribution over S
- Evaluation policy *h* 
  - Take the records of day *d*
  - *h* is the empirical distribution of these records
  - The value *V*(*h*) is the average CTR for these records
- Prediction errors on 100 most frequent queries as a function of clipping parameter *M*





• Records of K = 10 positions with DCG value



- Ground set  $E = \{1, \dots, L\}$  of L items
- A list is a *K*-permutation of *E*, which is an element of

Setting

$$\prod_{K} E = \{(a_1, \dots, a_K): a_1, \dots, a_K \in E; a_i \neq a_j, i \neq j\}$$

- Context set *X*
- $\overline{w}(a, k|x)$ : the expected CTR of putting item a in position k under context x
- A policy  $\pi$  is a conditional probability distribution of a list given context  $x: \pi(\cdot | x)$
- The reward of list A

 $f(A,w) = \sum_{k=1}^{K} w(a_k,k)$ 

- The value of a policy  $V(\pi) = \mathbb{E}_{x}[\mathbb{E}_{A \sim \pi(\cdot | \chi)} f(A, \overline{w}(\cdot | \chi))]$
- At each time *t* 
  - the environment draws context  $x_t$  and click realizations  $w_t$
  - The learner observes  $x_t$  and selects  $A_t$  according to policy  $\pi$
  - The environment reveals  $(w_t(a_k^t, k))_{k=1}^K$
- Logged dataset:  $S = \{(x_t, A_t, w_t)\}_{t=1}^n$



- Item-Position Click Model (IP): •  $\overline{w}(a, k|x)$  depends on both item a and position k  $\widehat{V}_{IP}(h) = \frac{1}{|S|} \sum_{(x,A,w)\in S} \sum_{k=1}^{K} w(a_k, k) \min\left\{\frac{h(a_k, k|x)}{\widehat{\pi}(a_k, k|x)}, M\right\}$  $\pi(a, k|x) = \sum_{A} \pi(A|x) 1\{a_k = a\}$
- Rank-Based Click Model (RCTR):
  - $\overline{w}(a, k | x)$  only depends on position k

$$\hat{V}_{R}(h) = \frac{1}{|S|} \sum_{(x,A,w)\in S} \sum_{k=1}^{K} w(a_{k},k)$$

Position-Based Click Model (PBM):

- IVI
- The performance of list estimator deteriorates fast with more positions
- The IP estimator performs best

#### Analysis

**Proposition 1.**[Unbiased in a larger class of policies] Let  $\mathcal{H}_Y$  contains all policies such that  $\hat{V}_Y$  is unbiased, for any  $Y \in \{L, IP, I, PBM\}$ . Then  $\mathcal{H}_L \subseteq \mathcal{H}_{IP} \subseteq \mathcal{H}_I / \mathcal{H}_{PBM}$ .

**Proposition 2.** [Lower bias in estimating policy]  $\mathbb{E}_{S}[\hat{V}_{L}] \leq \mathbb{E}_{S}[\hat{V}_{IP}] \leq \mathbb{E}_{S}[\hat{V}_{I}]/\mathbb{E}_{S}[\hat{V}_{PBM}] \leq V(h)$ 

**Proposition 3.**[Policy optimization] Suppose  $\tilde{h}_Y$  is the best policy under  $\hat{V}_Y$ , for any  $Y \in \{L, IP, I, PBM\}$ . Then the lower bound on  $\tilde{h}_Y$  is at least as high as that on  $\tilde{h}_L$ .

#### Conclusions

 We propose various estimators for the expected number of clicks on lists generated by ranking policies that leverage the structure of click models

- Objective
  - To design statistically efficient estimators based on logged dataset for any ranking policy
- Challenge
  - The number of different lists is exponential in *K*

•  $\overline{w}(a,k|x) = \mu(a|x)p(k|x)$  $\widehat{V}_{PBM}(h)$  $\sum_{k=1}^{n} w(a_k, k) \min\left\{\frac{\langle p(\cdot | x), h(a_k, \cdot | x) \rangle}{\langle p(\cdot | x), \hat{\pi}(a_k, \cdot | x) \rangle}, M\right\}$ 

- We prove that our estimators are better than the unstructured list estimators, in the sense that they are less biased and have better guarantees for policy optimization
- Our estimators consistently outperform the list estimator in our experiments

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## Full Paper



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