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Online Influence Maximization under Linear Threshold Model

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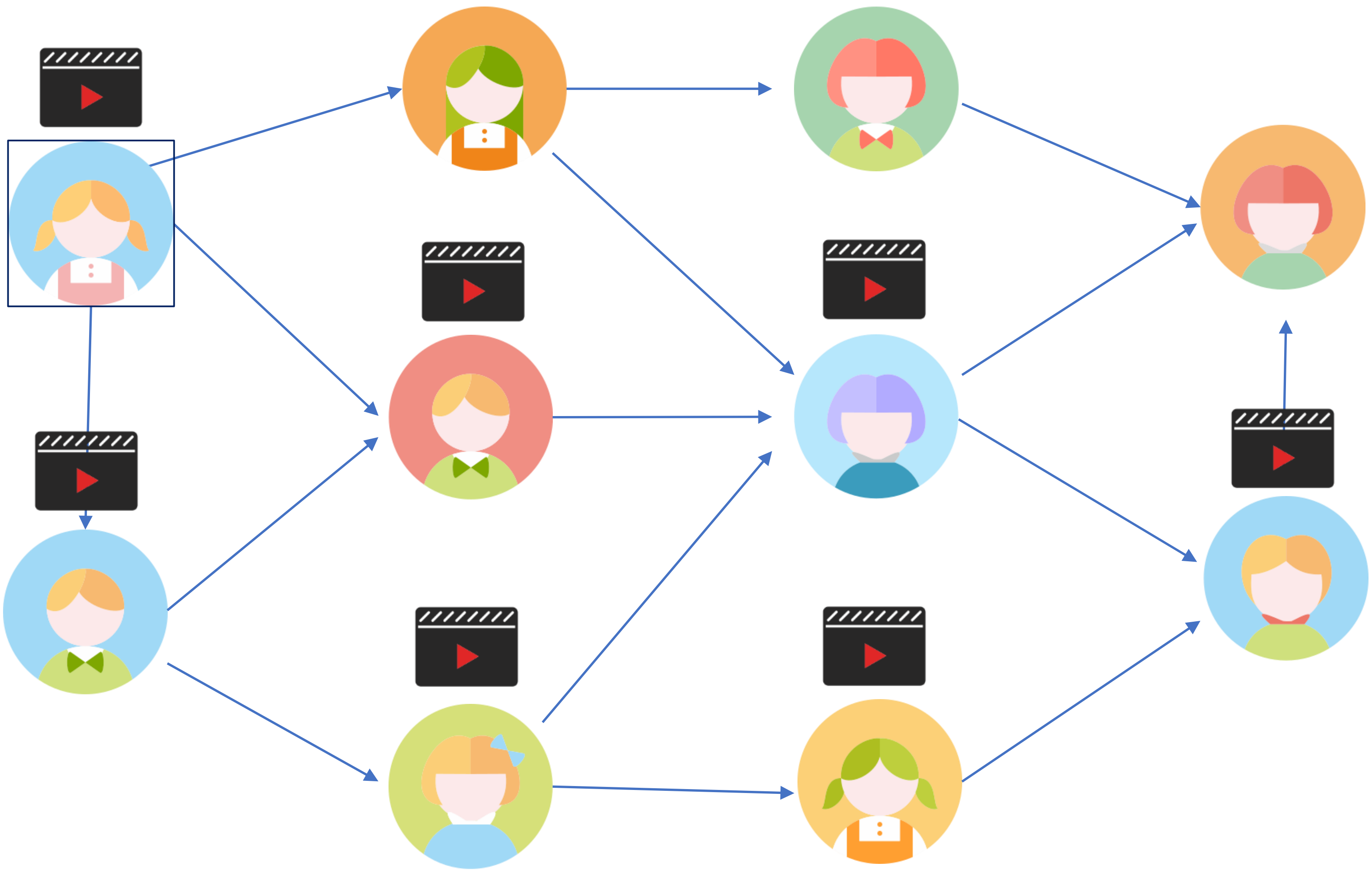
Let’s consider the scene where

the company wants to broadcast their products

or

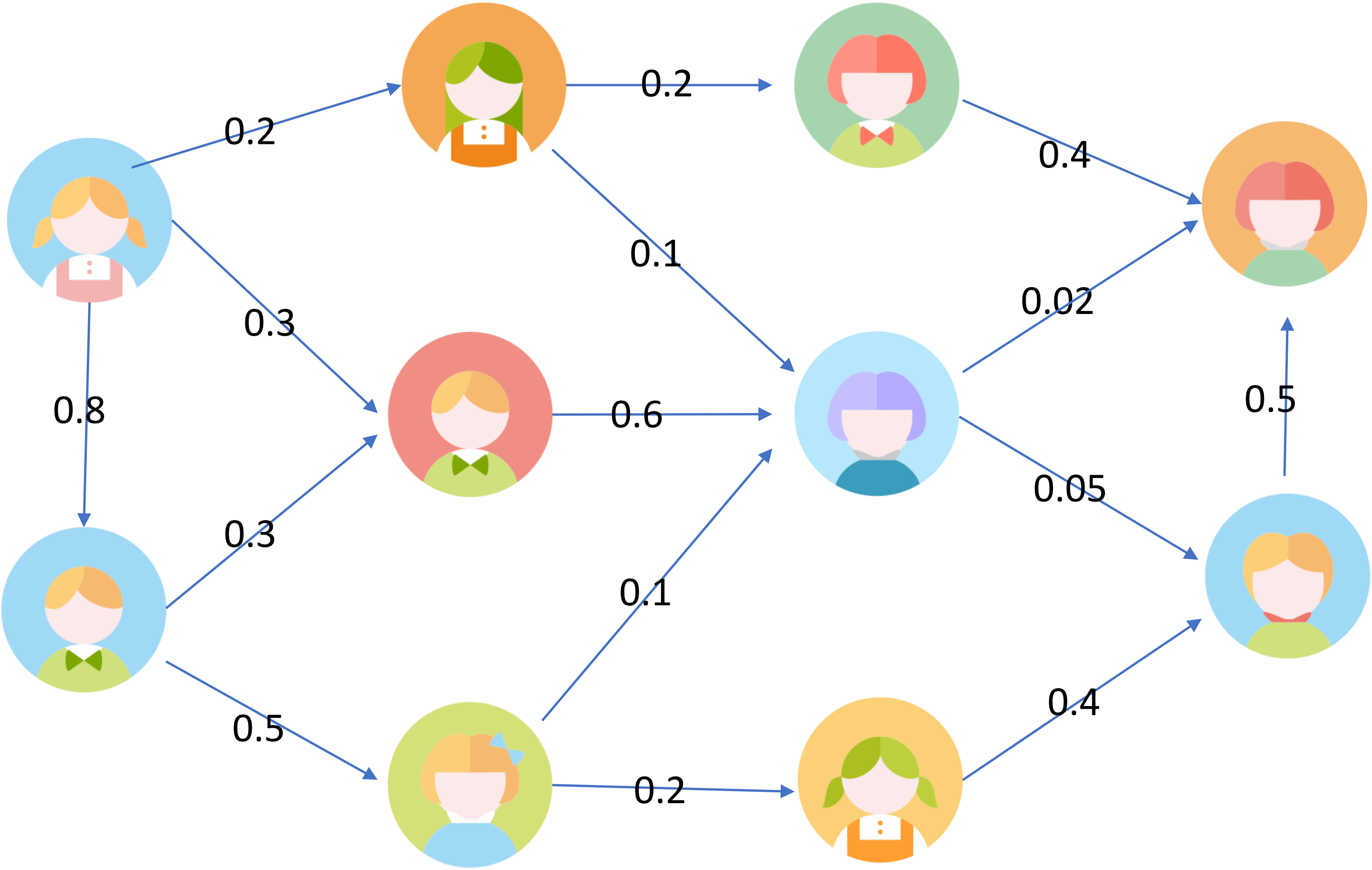
the government wants to promote their policies

With limited budget, the company choose where to place the advertisement thus the number of influenced users could be maximized?



When given influence probabilities between users:

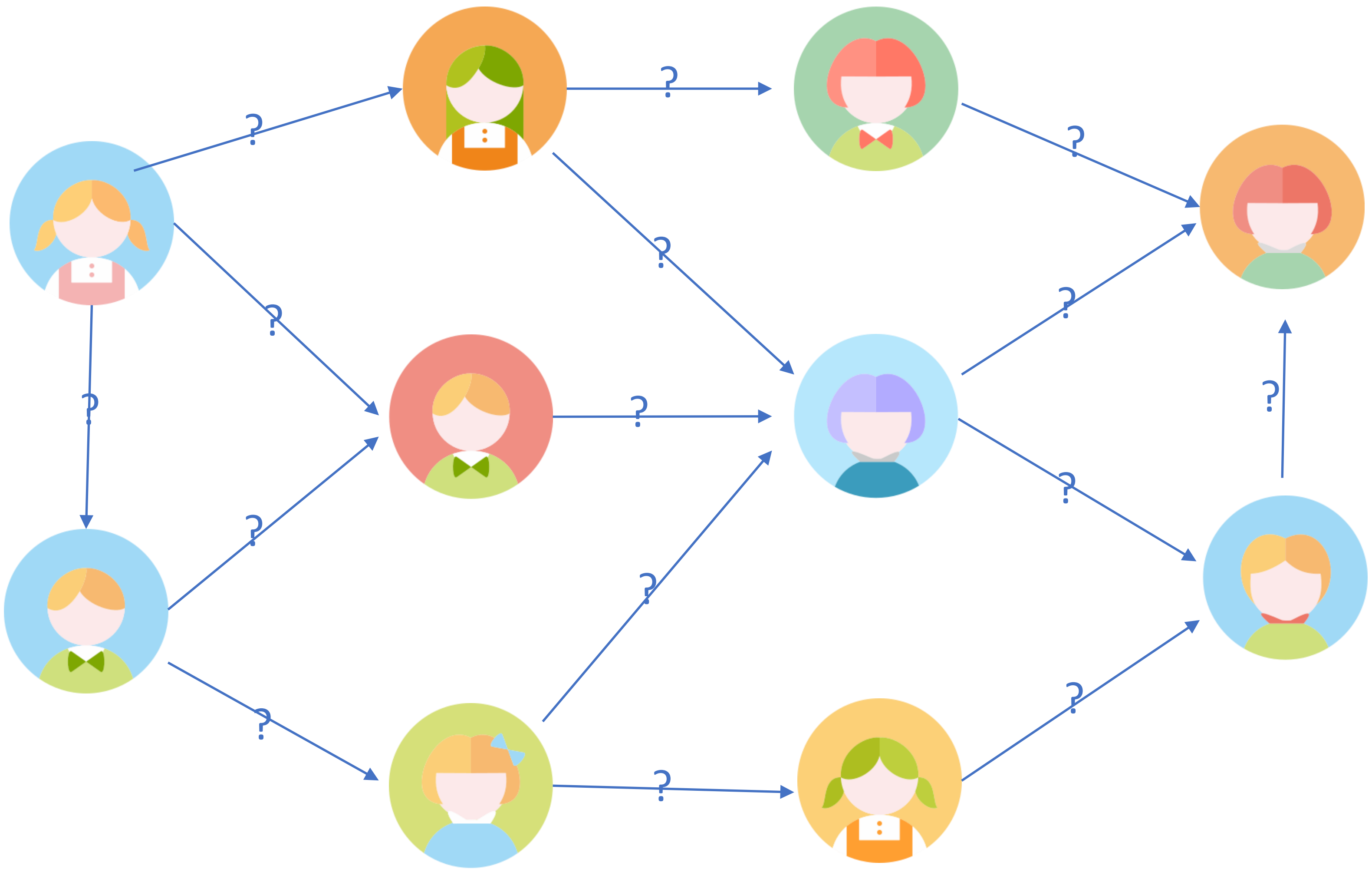
-This problem is well known as Influence Maximization (IM) problem



However,

It is not realistic to get those influence probabilities beforehand in real applications

So how to choose the optimal seed nodes with unknown influence probabilities?

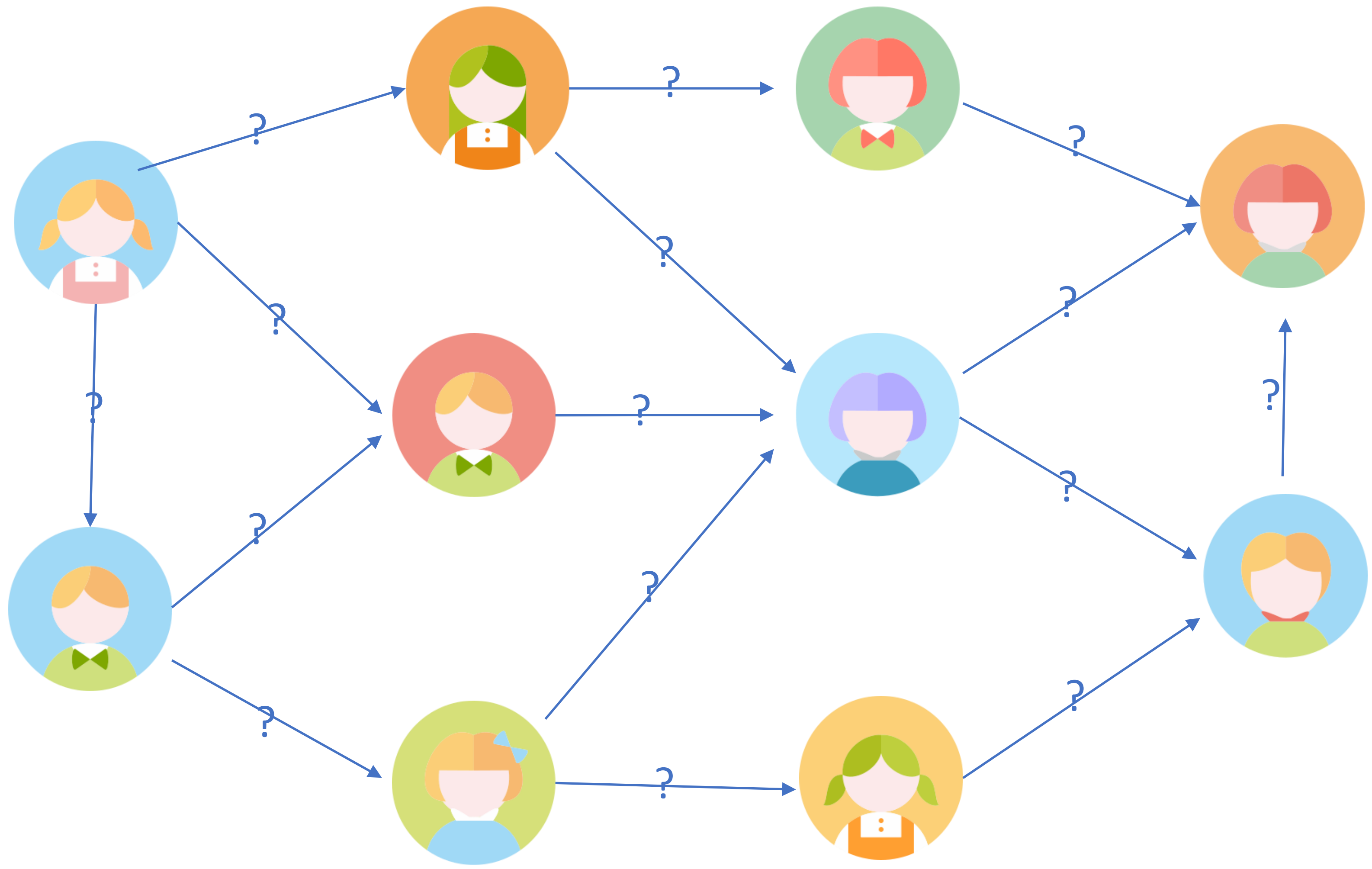


How about estimating the unknown parameters from the collected past observations?

-the log might have bias and deficient

-the estimates cannot adapt to any change in the social network

This motivates the Online Influence Maximization (OIM) framework



Online Influence Maximization (OIM) framework:

– learn unknown parameters during the trial-and-error process

for $t = 1,2,\ldots,T$:

·the agent chooses the seed set S_t according to its policy based on current knowledge

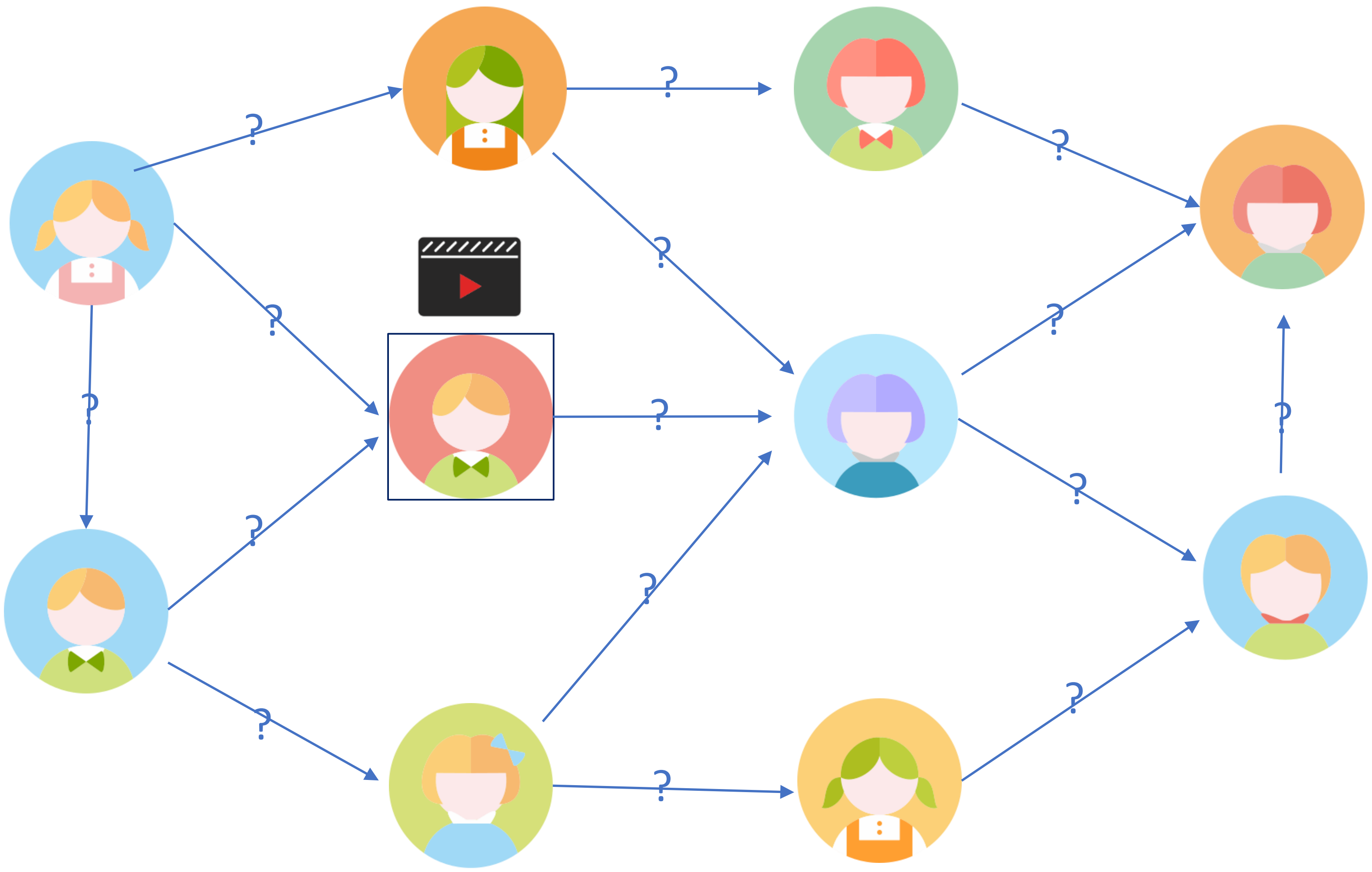
·then the agent observes the diffusion process and update its knowledge

end for

Objective: minimize the cumulative regret

$$R(T) = \mathrm{E}\left[\sum_{i=1}^{T} \eta \cdot \mathrm{opt}_w - r(S_t, w)\right]$$

$t = 1$: the agent chooses the red boy as the seed node



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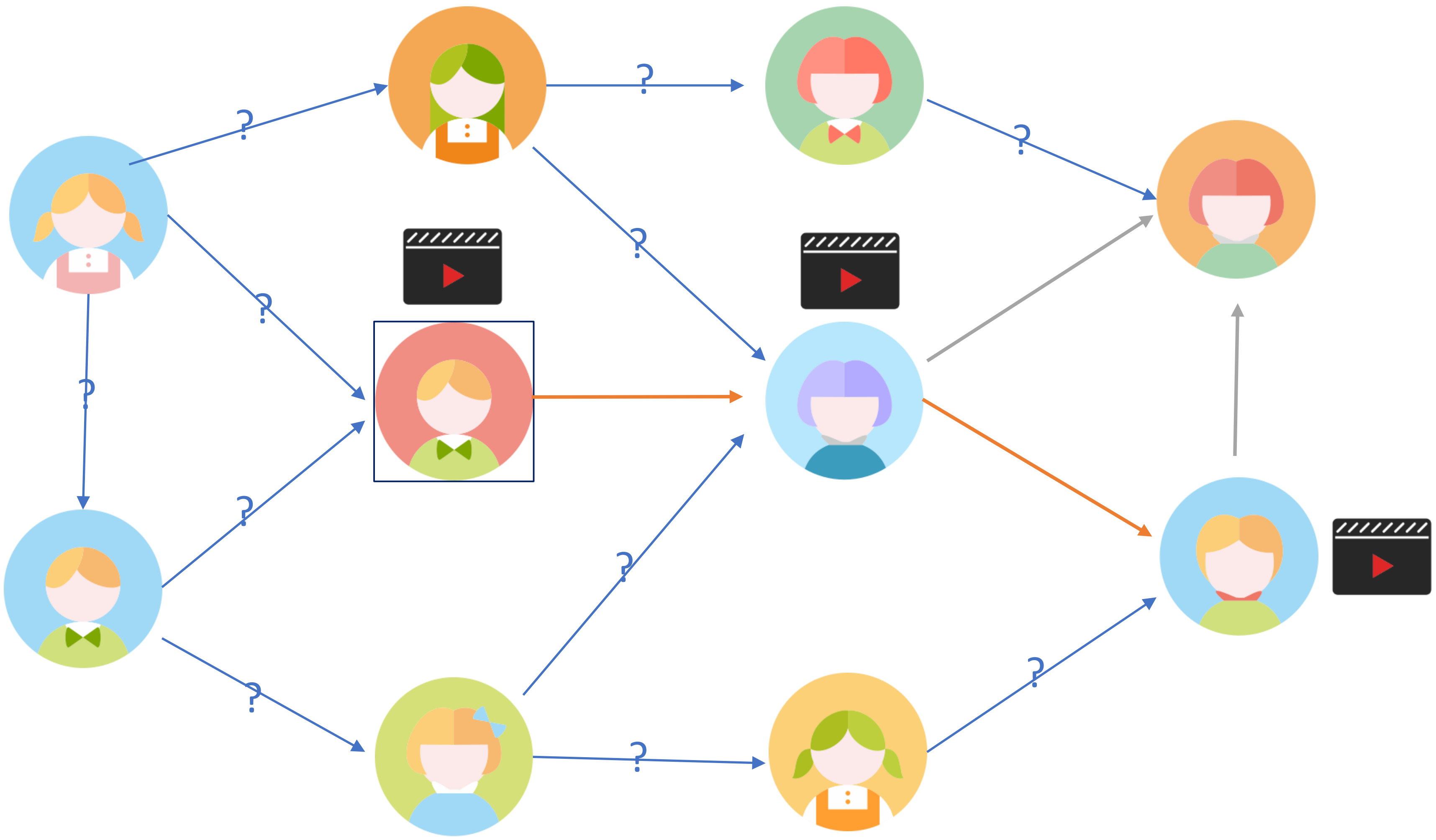

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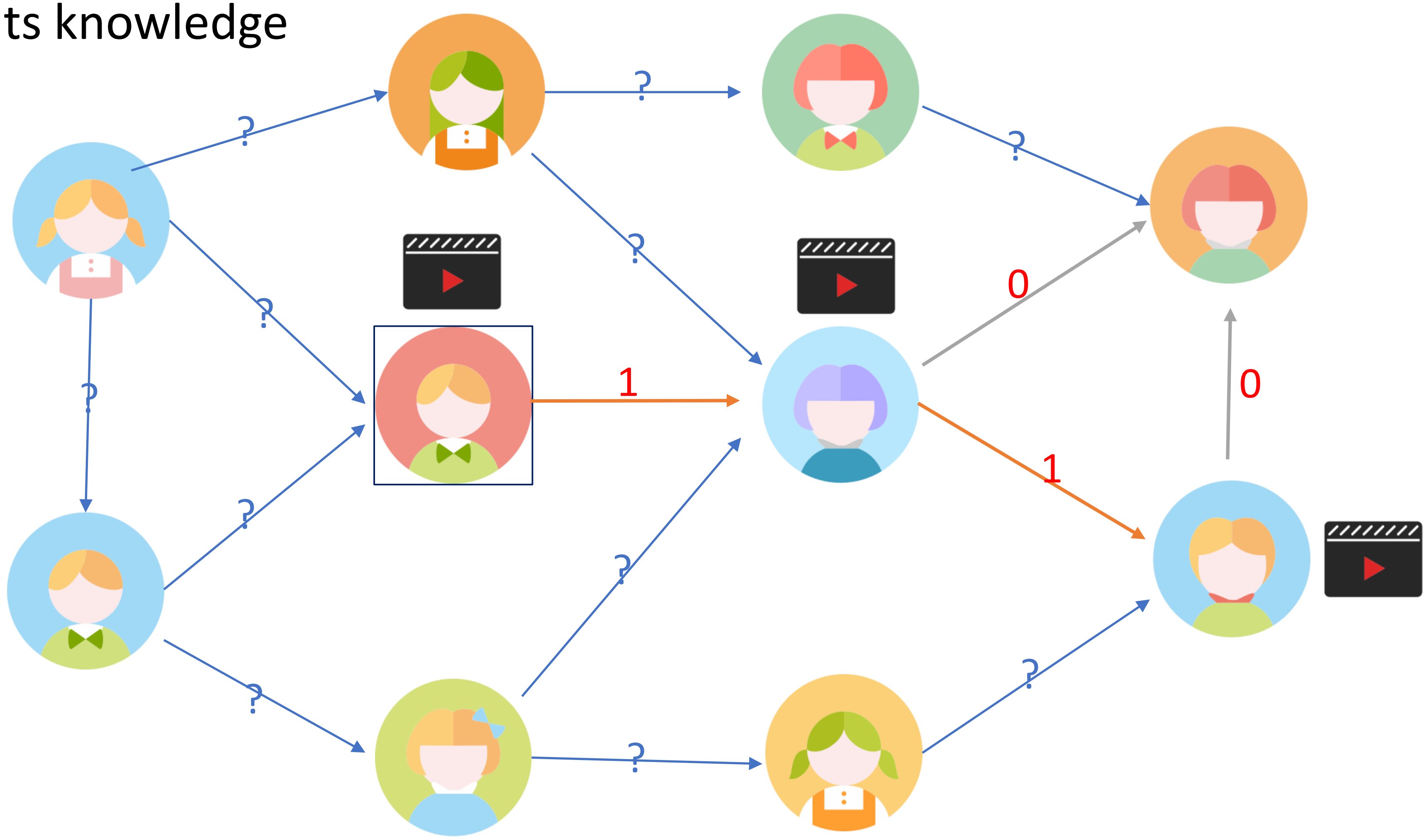
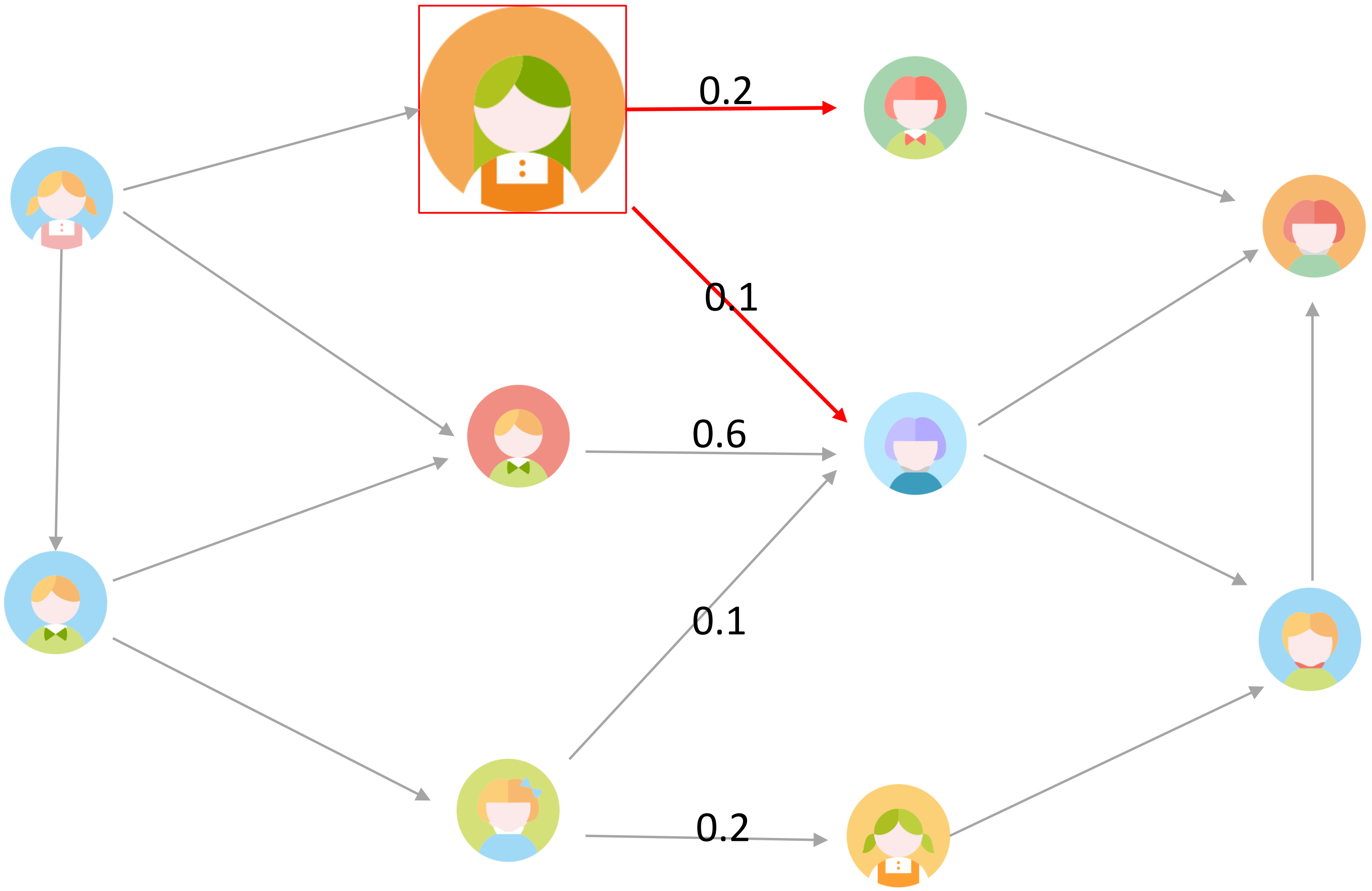


How to model the information diffusion in real world?

Independent cascade (IC) and linear threshold (LT) are two most common models

Independent Cascade (IC) Model



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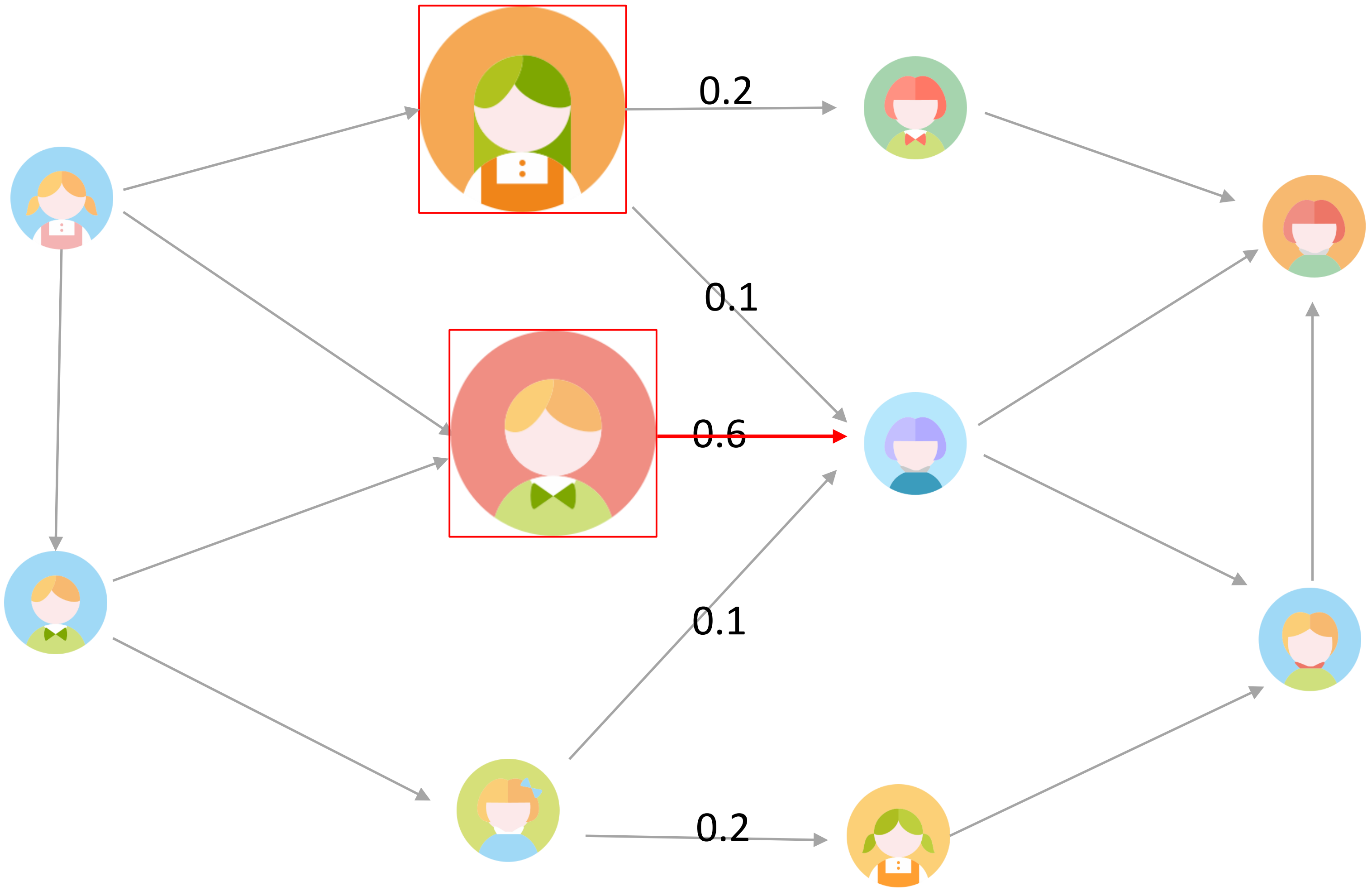

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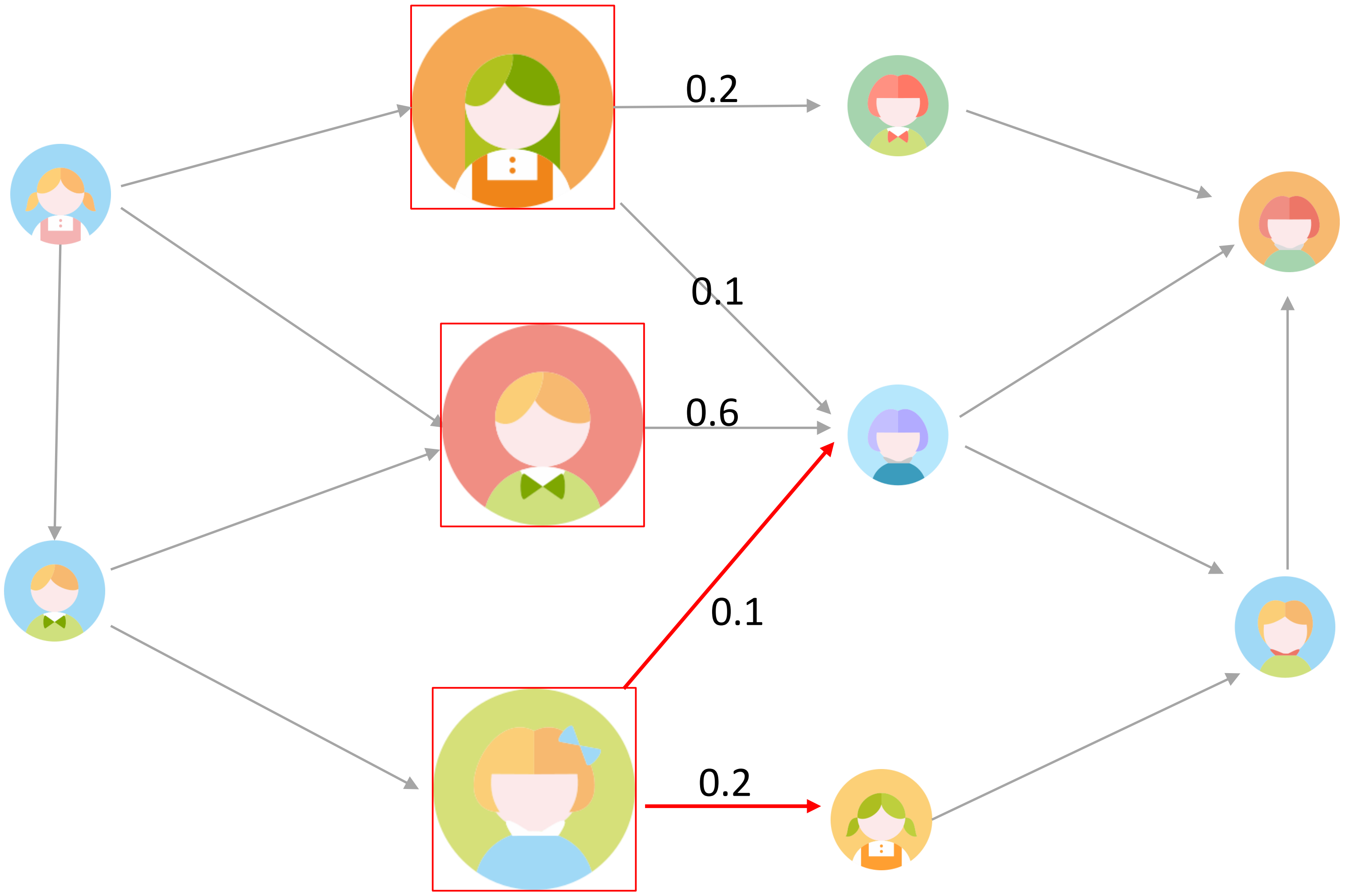
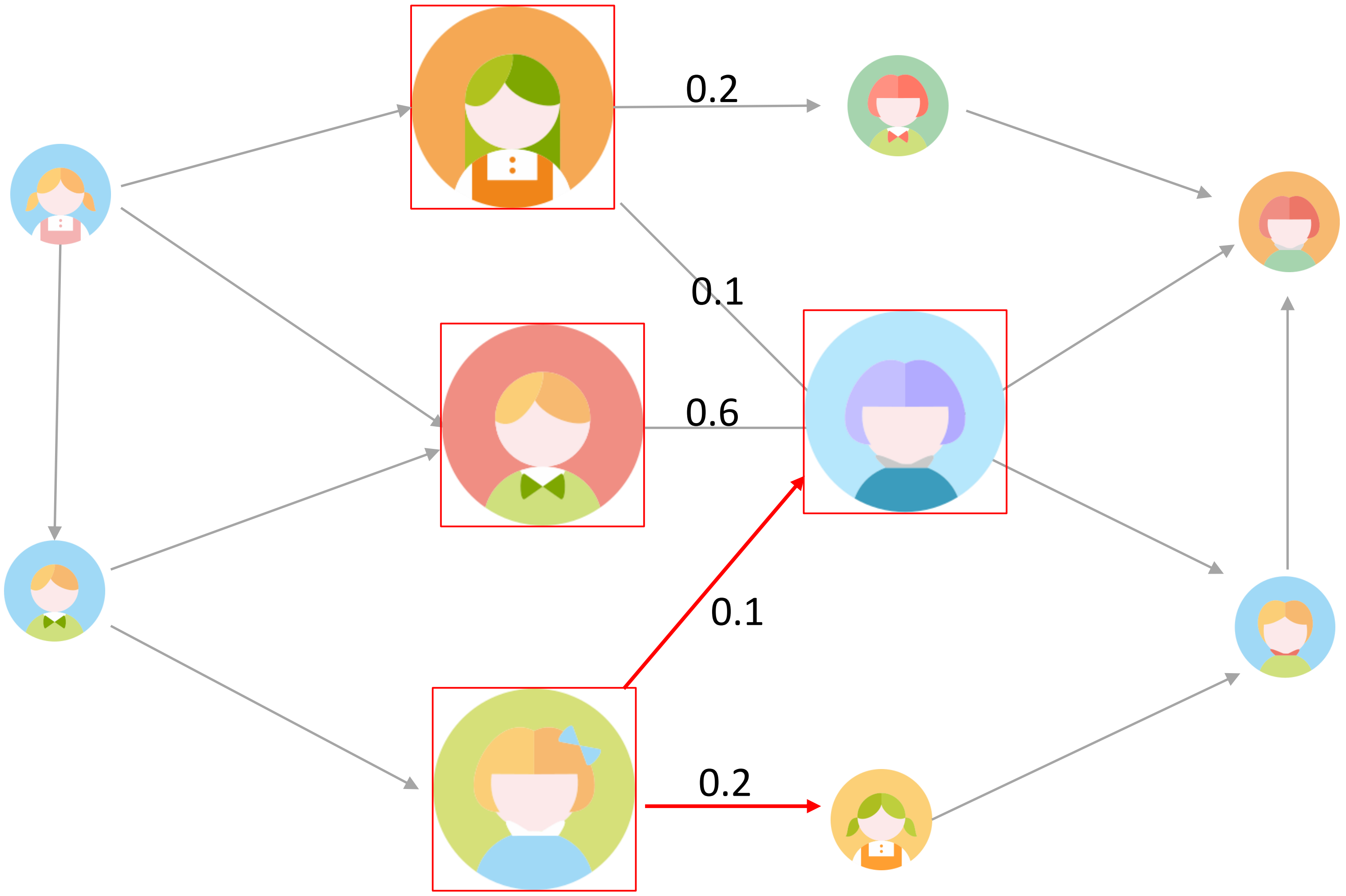


Independent Cascade (IC) Model



independence assumption

Previous OIM works mainly consider

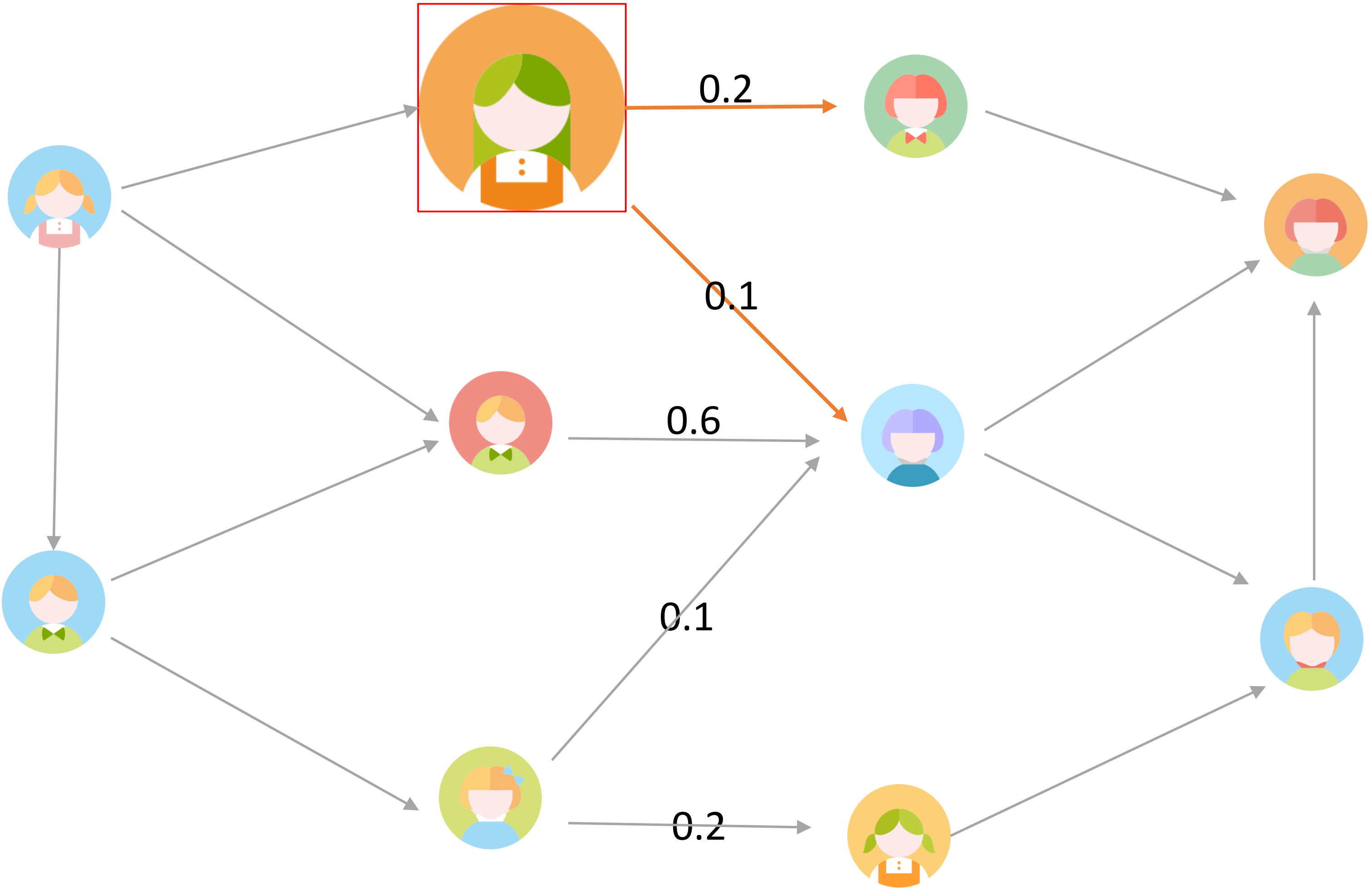
-IC model

-edge-level feedback

Which neighbor has successfully activated the node

Does this setting characterize the real information diffusion process well (from the perspective of the company/government) ?

Let's consider the common herd behavior in real life:



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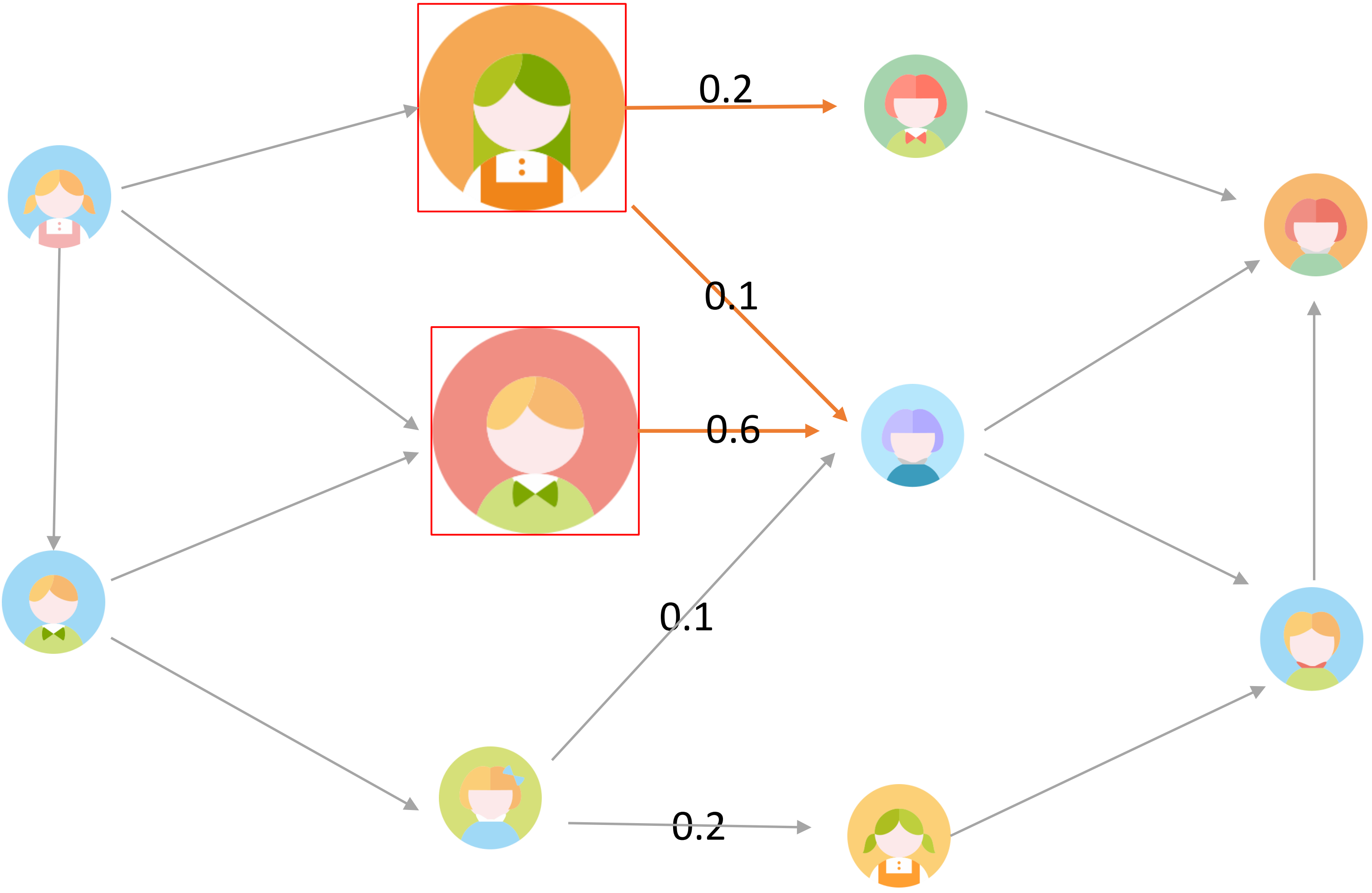

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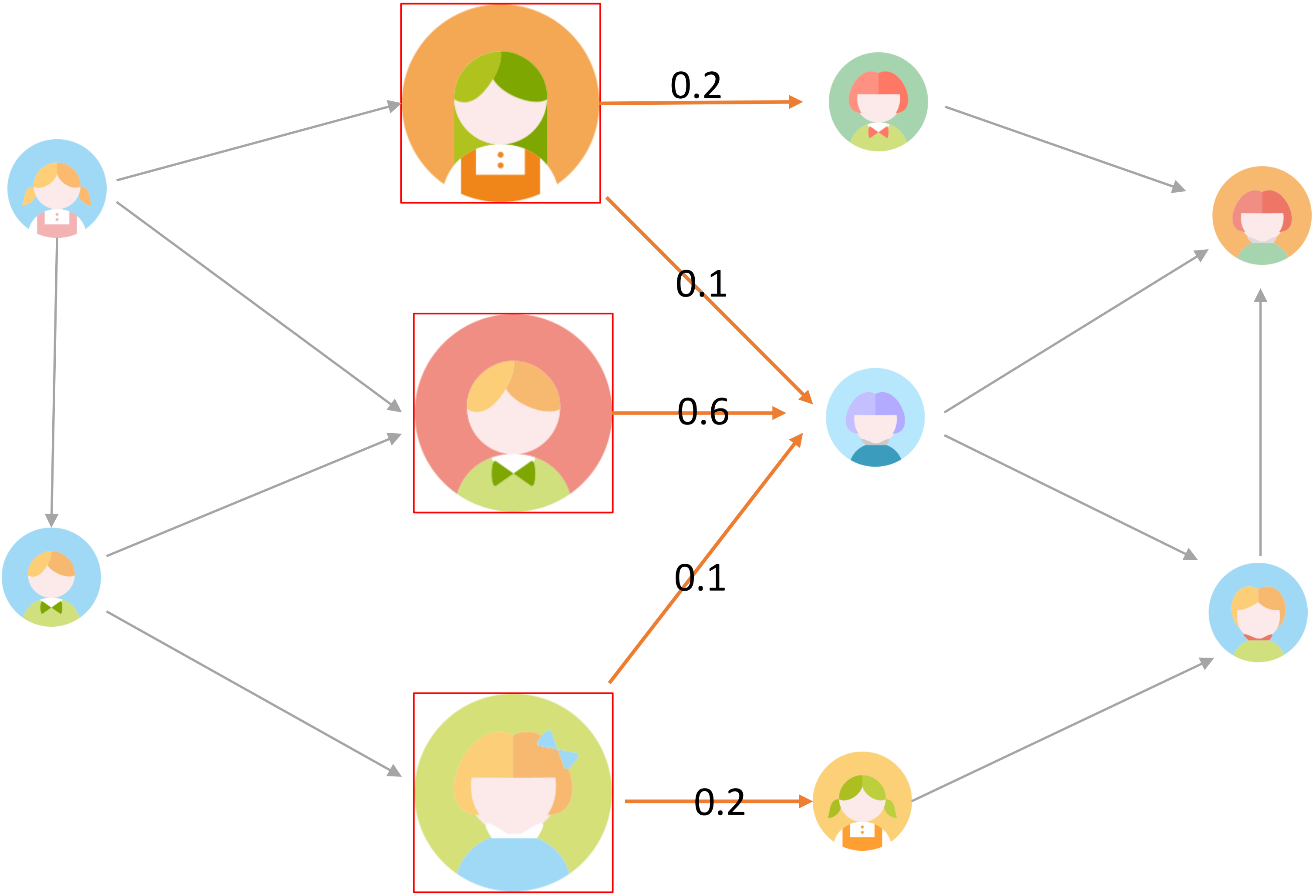

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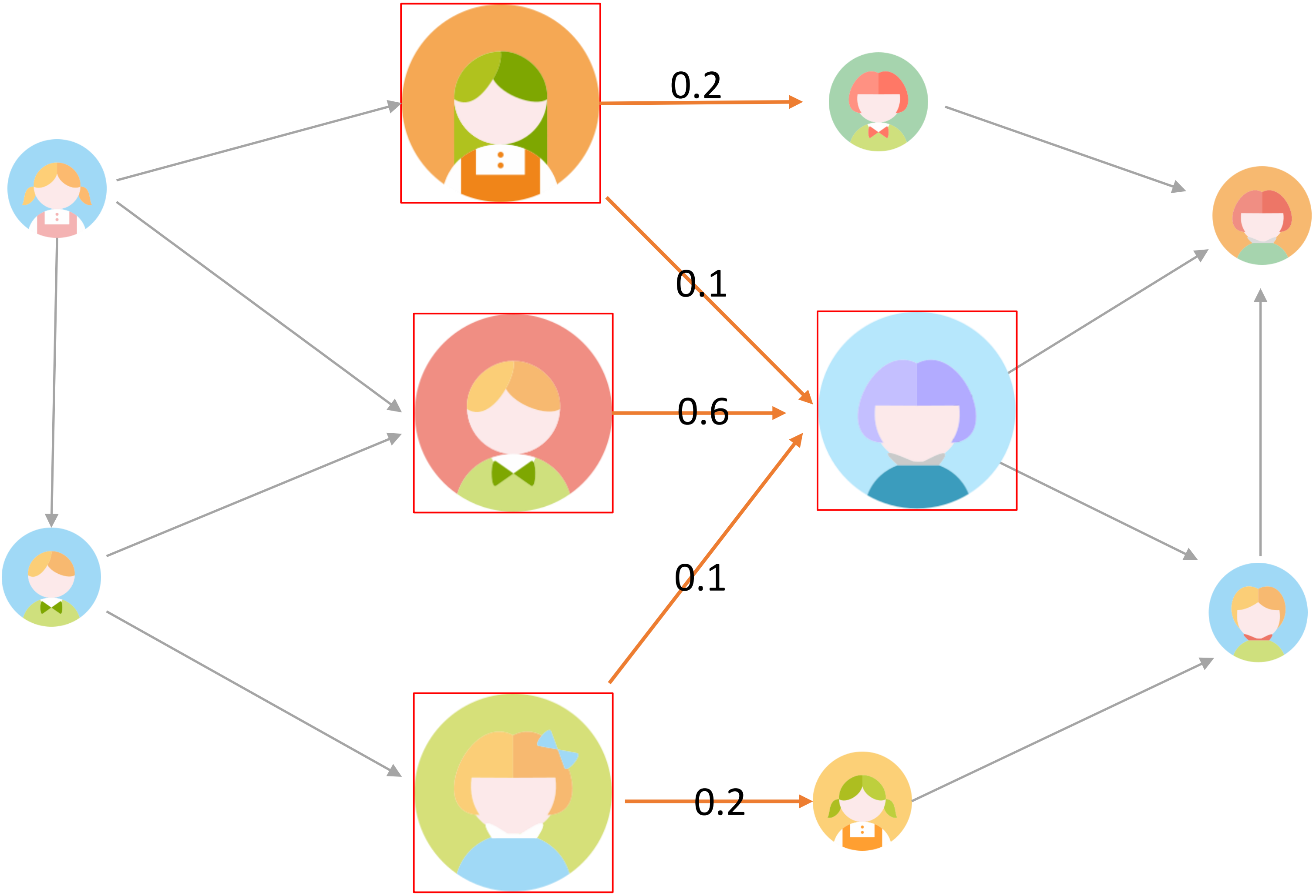
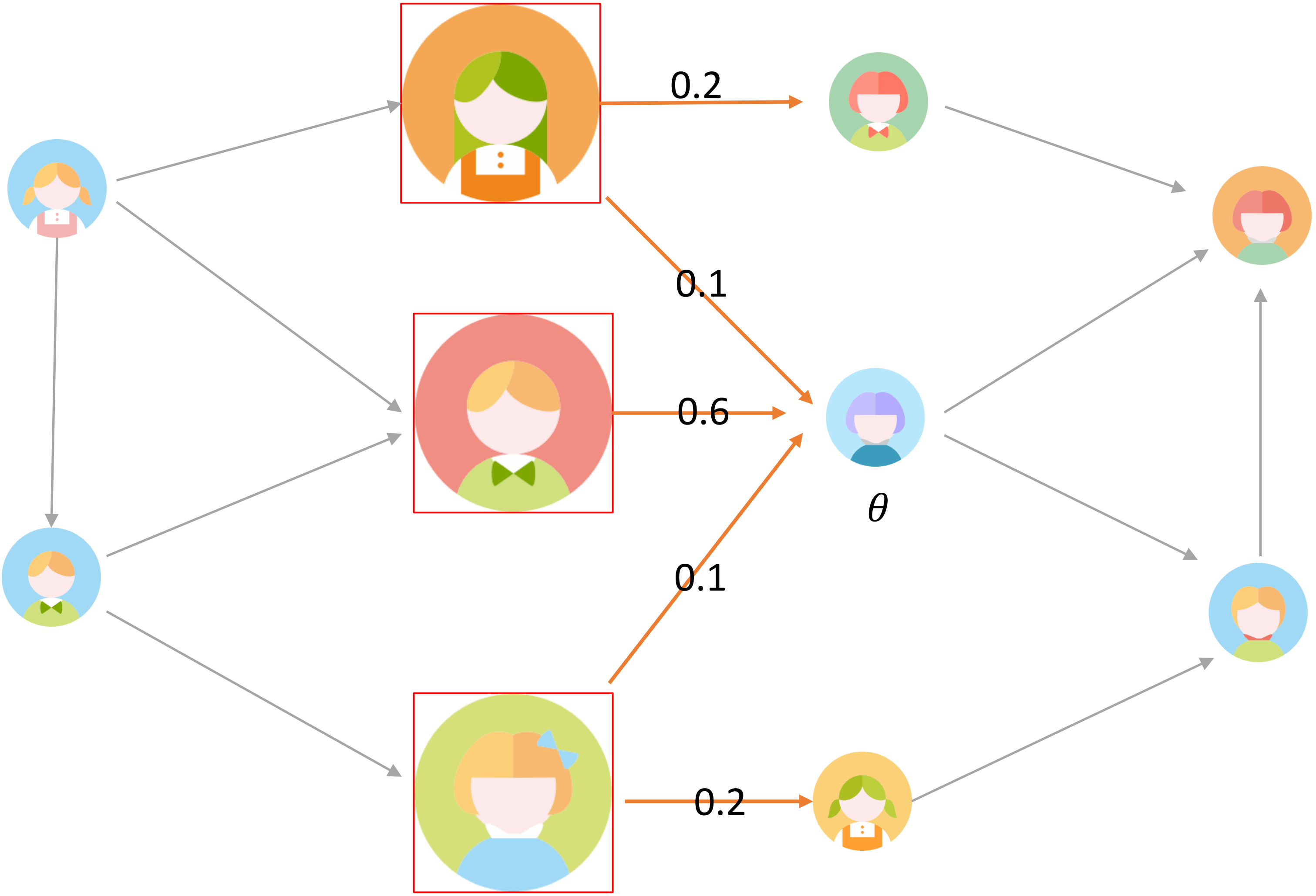


This phenomenon can be well described by the Linear Threshold (LT) Model



If the sum of weights from active in-neighbors $\geq \theta$: then this user is influenced

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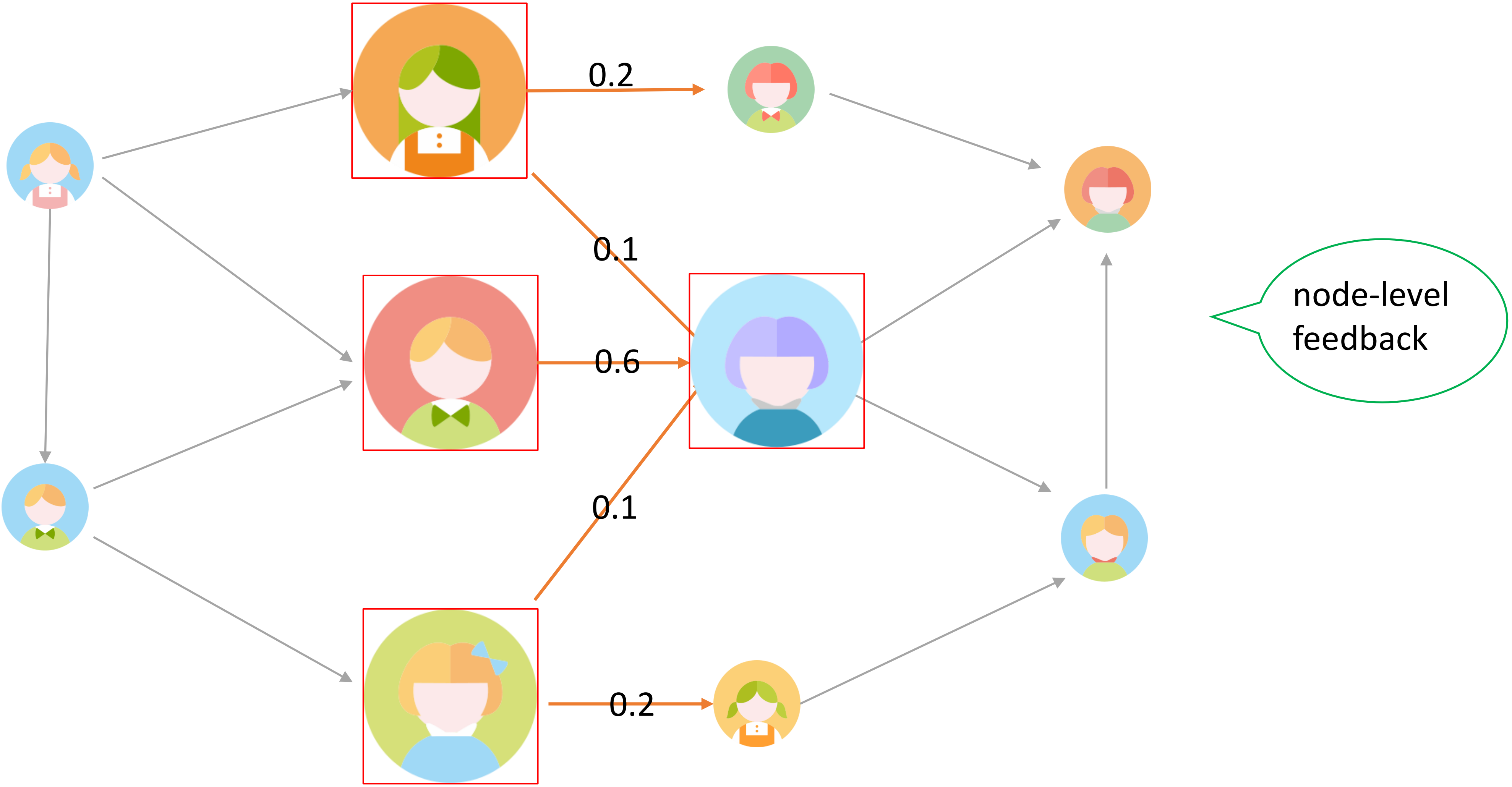


If the sum of weights from active in-neighbors $\geq \theta$: then this user is influenced

How to design the policy to solve the OIM problem?

The agent wants to

- learn unknown parameters as much as possible
- achieve as high influence spread as possible

Trade-off: exploration & exploitation

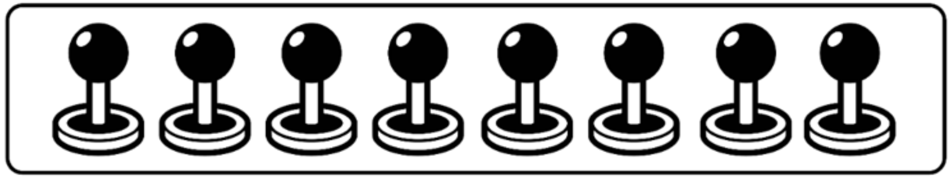
try those seed sets that the agent still does not know well to achieve potential high influence spread

focus on these seed sets which enjoy the maximum influence spread so far to get relatively high rewards

$\Downarrow$

Combinatorial Multi-armed Bandit with Probabilistically Triggered arms (CMAB-T)

CMAB-T framework

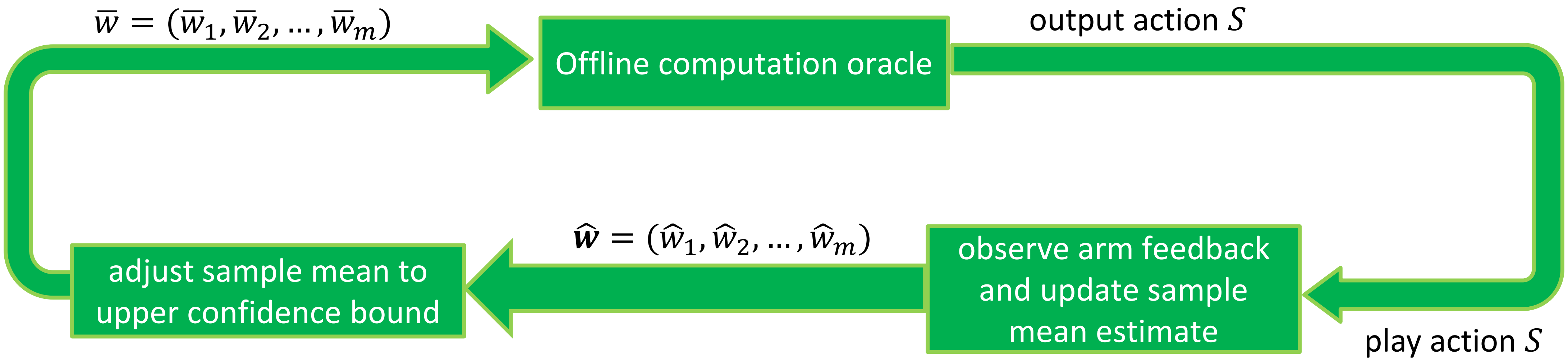


D_1 D_2 … D_m

- There are totally m base arms.

  Each arm i is associated with an unknown reward distribution D_i
- The agent selects a set of base arms, other arms may be probabilistically triggered
- The agent receive the reward of this action
- The agent has observations on the output of triggered arms and update its knowledge

Policy: Combinatorial Upper Confidence Bound (CUCB)



$$\bar{w}_i = \min\left\{\hat{w}_i + \sqrt{\frac{3\ln t}{2T_i}}, 1\right\}$$

key tradeoff between exploration and exploitation:

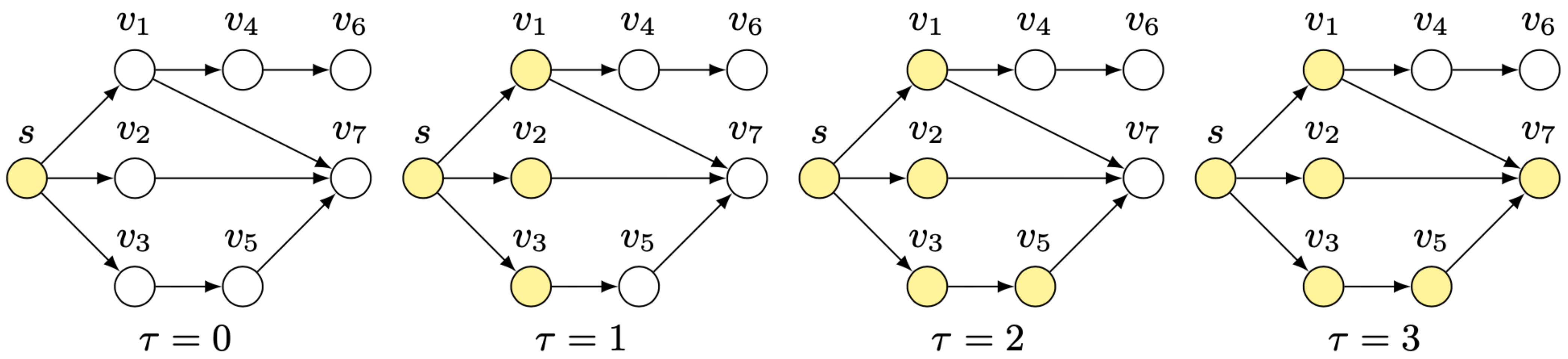
-if T_i is small, $\bar{w}_i$ is large (explore)

-otherwise $\bar{w}_i \to \hat{w}_i \to w_i$ (exploit)

T_i: # of times arm i is observed; initially 0

t: current round number; initially 1

How to solve the OIM problem under the LT model?



- Only group effect can be observed, how to estimate the unknown weight of each edge?
- The groups are also random and the observed group effect are correlated

How to solve the OIM problem under the LT model?

Algorithm 1 LT-LinUCB

Input: Graph $G = (V, E)$; seed set cardinality K; exploration parameter $\rho_{t,v} > 0$ for any t, v; offline oracle PairOracle

Initialize: $M_{0,v} \leftarrow I \in \mathbb{R}^{|N(v)| \times |N(v)|}$, $b_{0,v} \leftarrow 0 \in \mathbb{R}^{|N(v)| \times 1}$, $\hat{w}_{0,v} \leftarrow 0 \in \mathbb{R}^{|N(v)| \times 1}$ for any node $v \in V$

for $t = 1, 2, 3, \ldots$ **do**

  Compute the confidence ellipsoid $\mathcal{C}_{t,v} = \left\{ w'_v \in [0,1]^{|N(v)| \times 1} : \| w'_v - \hat{w}_{t-1,v} \|_{M_{t-1,v}} \le \rho_{t,v} \right\}$ for any node $v \in V$

  Compute the pair (S_t, w_t) by PairOracle with confidence set $\mathcal{C}_t = \{\mathcal{C}_{t,v}\}_{v \in V}$ and seed set cardinality K

  Select the seed set S_t and observe the feedback

  // Update

  for node $v \in V$ **do**

    Initialize $A_{t,v} \leftarrow 0 \in \mathbb{R}^{|N(v)| \times 1}$, $y_{t,v} \leftarrow 0 \in \mathbb{R}$

    Uniformly randomly choose $\tau \in \{\tau' : \tau_{t,1}(v) \le \tau' \le \tau_{t,2}(v) - 1\}$

    if v is influenced and $\tau = \tau_{t,2}(v) - 1$ **then**

      $A_{t,v} = \chi\left(E_{t,\tau}(v)\right), \quad y_{t,v} = 1$

    else if $\tau = \tau_1(v), \ldots, \tau_2(v) - 2$ or $\tau = \tau_2(v) - 1$ but v is not influenced **then**

      $A_{t,v} = \chi\left(E_{t,\tau}(v)\right), \quad y_{t,v} = 0$

    end if

    $M_{t,v} \leftarrow M_{t-1,v} + A_{t,v} A_{t,v}^\top, \quad b_{t,v} \leftarrow b_{t-1,v} + y_{t,v} A_{t,v}, \quad \hat{w}_{t,v} = M_{t,v}^{-1} b_{t,v}$

  end for

end for

How to solve the OIM problem under the LT model?

·Through detailed analysis of the information diffusion and breaking the complex correlation of LT model

·Group observation modulated (GOM) bounded smoothness property is proved to analyze the difference of the influence spread

How to solve the OIM problem under the LT model?

Theorem 1. *(GOM bounded smoothness) For any two weight vectors* $w, w' \in [0,1]^m$ *with* $\sum_{u \in N(v)} w(e_{u,v}) \leq 1$, *the difference of their influence spread for any seed set* S *can be bounded as*

$$|r(S, w') - r(S, w)| \leq \mathbb{E}\left[\sum_{v \in V \setminus S} \sum_{u \in V_{S,v}} \sum_{\tau = \tau_1(u)}^{\tau_2(u)-1} \left| \sum_{e \in E_\tau(u)} (w'(e) - w(e)) \right| \right], \qquad (6)$$

where the definitions of $\tau_1(u), \tau_2(u)$ *and* $E_\tau(u)$ *are all under weight vector* w, *and the expectation is taken over the randomness of the thresholds on nodes.*

This theorem connects the reward difference with weight differences on the distilled observations, which are also the information used to update the algorithm

This is the first OIM result under LT model

Our LT-LinUCB(LT)	IMLinUCB(IC)	CUCB(IC)
$\tilde{O}(n^{7/2}\sqrt{m} \cdot \sqrt{T})$	$\tilde{O}(nm^{3/2} \cdot \sqrt{T})$	$\tilde{O}(nm \cdot \sqrt{T})$

n is the number of nodes and m is the number of edges

Which may due to less observed information under LT model.

Thanks!